

# **GRACE**

Gravity Recovery and Climate Experiment

**JPL Level-2 Processing Standards Document**

**For Level-2 Product Release 05**

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# DOCUMENT CHANGE RECORD

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## I DOCUMENT DESCRIPTION

### I. 1 PURPOSE OF THE DOCUMENT

This document serves as a record of the processing standards, models & parameters adopted for the generation of the Level-2 gravity field data products by the GRACE Science Data System component at NASA The Jet Propulsion Laboratory of the California Institute of Technology (JPL). This document is issued once for every release of Level-2 data products generated by JPL. The release number refers to the field *RL* in the generic Level-2 product name (see *Product Specification Document* or *Level-2 User Handbook*)

*PID-2\_YYYYDOY-YYYYDOY\_RL*

This document uses in its title the release number *RL* for the specific product release whose processing standards are described herein.

This document may be used in conjunction with:

1. GRACE Product Specification Document (327-720)
2. GRACE Level-2 User Handbook (327-734)
3. GRACE CSR L-2 Processing Standards Document (327-742)
4. GRACE GFZ L-2 Processing Standards Document (327-743)
5. GRACE AOD1B Product Description Doc (327-750, GR-GFZ-AOD-0001)

### I. 2 DOCUMENT CHANGE HISTORY

This document has been previously issued for the Level-2 data product releases as listed in the change log earlier in this document. The principal changes since the previous issue of this document are described in the remainder of this document.

## II ORBIT DYNAMICS MODELS

### II.1 EQUATIONS OF MOTION

The equations of motion for both GRACE satellites are identical in mathematical form. In the remainder of this chapter, the equations will be provided for a single Earth orbiting satellite, with the understanding that the same equations apply to both GRACE satellites. Where appropriate, the parameters or conditions unique to each satellite will be specified.

In the inertial frame

$$\ddot{\vec{r}} = \vec{f}_g + \vec{f}_{ng} + \vec{f}_{emp}$$

where the subscript “g” denotes gravitational accelerations; “ng” denotes the acceleration due to the non-gravitational or skin forces; and “emp” denotes certain empirically modeled forces designed to overcome deficiencies in the remaining force models.

#### II.1.1 Independent Variable (Time Systems)

The independent variable in the equations of motion is the TDT (Terrestrial Dynamic Time). The relationship of this abstract, uniform time scale to other time systems is well known. The table below shows the relationship between various time systems and the contexts in which they are used.

System	Relations	Notes	Standards
TAI	Fundamental time system	International Atomic Time	na
UTC	TAI = UTC + n1 (Time-tag for saving intermediate products)	n1 are the Leap Seconds	Tables from USNO
TDT	TDT = TAI + 32.184 s	This is the independent variable for integration. Distinction between TDB & TDT is ignored.	IAG 1976 recommendations
GPS	GPS = UTC + n2 (basis for the time-tagging of GRACE Observations)	n2 are Leap Seconds since Jan 6, 1980	Time-tags in sec since 1200 Jan 01, 2000 GPS Time.

### II.1.2 Coordinate System

The fundamental reference frame for the mathematical model is the non-rotating, freely-falling (inertial) reference frame with the origin defined as the center of mass of the Earth. The Inertial and Earth-fixed reference frames, and their relative orientations and associated standards are further described in the chapter on Earth Kinematics.

## II. 2 GRAVITATIONAL FORCES

The gravitational accelerations are the sum of direct planetary perturbations and the geopotential perturbations. The vector of direct planetary perturbations is evaluated using the planetary ephemerides. The geopotential itself is represented in a spherical harmonic series with time-variable coefficients, to a specified maximum degree and order, and accelerations are computed by evaluating the Earth-fixed gradient of the geopotential. The accelerations are then rotated (after summation with the non-gravitational accelerations) to inertial frame for the integration of equations of motion. In general,

$$\vec{f}_g = {}_{3 \times 3} M_{ef}^{in}(P, N, R) \vec{f}_g^{ef}$$

The 3x3 rotation matrix M, which depends on Earth Precession, Nutation & Polar Motion is described in the chapter on Earth Kinematics.

Contributions to the spherical harmonic coefficients of the geopotential, and the associated implementation & standards are now compiled. The geopotential at an exterior field point, at time t, is expressed as

$$U_s(r, \varphi, \lambda; t) = \frac{GM_e}{r} + \frac{GM_e}{r} \sum_{l=2}^{N_{\max}} \left( \frac{a_e}{r} \right)^l \sum_{m=0}^l \bar{P}_{lm}(\sin \varphi) [\bar{C}_{lm}(t) \cos m\lambda + \bar{S}_{lm}(t) \sin m\lambda]$$

where r is the geocentric radius, and  $(\varphi, \lambda)$  are geographic latitude and longitude, respectively, of the field point.

The model used for propagation of the equations of motion of the satellites is called the Background Gravity Model. This concept, and its relation to GRACE estimates, is described further in the *Level-2 User Handbook*. The details of the background gravity model are provided here.

### II.2.1 Mean Geopotential & Secular Changes

Parameter	Value	Remarks
$GM_e$	3.986004415E+14 m <sup>3</sup> /s <sup>2</sup>	<i>IERS2000 Standards</i>

$a_e$	6378136.3 m	
$N_{\max}$	180	<b>GIF48</b> is background model
<p><u>Note 1:</u> The normalization conventions are as defined in IERS96, Chapter 6, Eqs 2-3.</p> <p><u>Note 2:</u> The implementation of computation of spherical harmonics &amp; its derivatives is as described in (Lundberg &amp; Schutz, 1988).</p> <p><u>Note 3:</u> Note that the degree 1 terms are identically zero when the origin of the coordinate system is the center of mass of the Earth</p>		

### II.2.2 Solid Earth Tides

Solid Earth tidal contribution to the geopotential are computed approximately as specified in Chapter 7, *IERS92 Conventions*. Corrections to specific spherical harmonic coefficients are computed and added to the mean field coefficients.

Model	Notes	
Planetary Ephemerides	DE-405	
Frequency Independent Terms	Degree 2 & 3 – expression in Eq. (1), Ch.6, <i>IERS2000</i> .	Constants from <i>IERS2003</i> are used.
	Ellipticity contributions from Degree 2 tides to Degree 4 terms	<i>IERS2003</i>
	External Potential Love Numbers	<i>IERS2003</i>
	Anelasticity Contributions	<i>IERS2003</i>
Frequency Dependent Terms	Tidal corrections to (2,1)	<i>IERS2003</i>
	Anelasticity Contributions	<i>IERS2003</i>
Permanent Tide in $\bar{C}_{20}$	4.173E-9	Removed from these contributions (is implicitly included in value of C20)

### II.2.3 Ocean Tides

The ocean tidal contributions to the geopotential are computed as specified in JPL Interoffice Memorandum “Convolution Formulism for the Ocean Tide Potential” by S. Desai, 4 March 2005. Corrections to specific spherical harmonic coefficients of arbitrary (selectable) degree and order are computed and added to the mean field coefficients.

Model	Description	Notes
Convolution Weights	Derived from GOT4.7 (monthly, fortnightly, diurnal, semidiurnal) and	



	SCEQ (Semi-annual and Annual)	
Expansion	Complete to degree 90	

### II.2.4 Tabular Atmosphere & Oceanic Variability

The non-tidal variability in the atmosphere and oceans is removed through using the AOD1B product. This product is a combination of the ECMWF operational atmospheric model and a barotropic ocean model driven with this atmospheric model. For JPL RL05, we use the AOD1B RL05, based upon ECMWF (as usual) and the baroclinic Dresden OMCT model with mass runoff constrained to zero. The details of this product and its generation are given in the *AOD1B Description Document (GRACE 327-750)*.

This component of the geopotential is ingested as 6 hourly time series to degree and order 100. The value of the harmonics at intermediate epochs is obtained by linear interpolation between the bracketing data points. Prior to its use, an estimate of the atmospheric S2 tidal effects on  $\bar{C}_{22}$  and  $\bar{S}_{22}$  are removed from the AOD1B product. This estimate is simply the difference of the TEG4 multi-satellite estimate of this tidal harmonic and the altimetric determination of this harmonic from the CSR 4.0 tidal model. In this way, the combination of the atmospheric and oceanic S2 tidal effects on the (2,2) harmonics are modeled using the ocean tide model.

### II.2.5 Rotational Deformation (Pole Tide)

The rotation deformation forces are computed as additions to spherical harmonic coefficients  $\bar{C}_{21}$  and  $\bar{S}_{21}$ , from an elastic Earth model, as specified in Chapter 6, IERS 96 Standards.

Model	Description	Notes
Elastic Earth Model Contribution to C21 & S21	Scaled difference between epoch pole position and mean pole. See Chapter III (Earth Kinematics) for values and linear variation model for the mean pole.	
Polar Motion	Tabular input	
Mean Polar Motion & Rates	Linear trend	<i>IERS2003</i>
Constant Parameters	Scale factor = $-1.333 \times 10^{-9}$ / arcsec	$K_2 = 0.3077 + i0.0036$
Anelasticity	Included, <i>IERS2003</i>	

### II.2.6 N-Body Perturbations

Unlike the geopotential accelerations, the perturbations due to the Sun, Moon and all the planets are directly computed as accelerations acting on the spacecraft. The direct effects of the objects on the satellite are evaluated using point-mass attraction formulas. The indirect effects due to the acceleration of the Earth by the planets are also modeled as point-mass interactions. However, for the Sun & the Moon, the indirect effects include the interaction between a point-mass perturbing object and an oblate Earth – the so-called Indirect J2 effect.

Model	Description	Notes
Third-Body Perturbation	Direct & Indirect terms of point-mass 3 <sup>rd</sup> body perturbations	
Indirect J2 Effect	Moon only	
Planetary Ephemerides	DE-421	

### ***II.2.7 General Relativistic Perturbations***

The general relativistic contributions to the accelerations are computed as specified in Chapter 10 of the IERS2000 Standards.

## **II. 3 NON-GRAVITATIONAL FORCES**

The nominal approach is to use the GRACE accelerometer data to model the non-gravitational forces acting on the satellite.

The model used is:

$$\vec{f}_{ng} = q \otimes \left[ \vec{b} + {}_{3 \times 3} E \vec{f}_{acc} \right]$$

where the q/operator represents rotations to inertial frame using the GRACE Attitude Quaternion product; b represents an empirical bias vector; and the 3x3 matrix E contains the scale factors along the diagonal, and no cross-coupling terms in the off-diagonal, that is, the matrix we model is diagonal at present.

The bias vector & scale matrix operate on the GRACE Accelerometer observation product, and are estimatable parameters. Bias rate is estimated for the X and Y components starting in 2010 to reflect thermal variations.

## **II. 4 EMPIRICAL FORCES**

For this product release, no empirical accelerations are modeled or estimated.

**II. 5 NUMERICAL INTEGRATION**

The DIVA variable step/variable order integrator of Krogh (1973) is implemented.

Model	Description	Notes
Dependent Variables		
1. Equations of motion (position/velocity for each satellite)		
2. State Transition Matrix (position/velocity mapping terms only)		
Formulation	Cowell Formulation	
Order	7	
Step-Size	Variable, nominally 5 second	Varied with 1.E-12 tolerance for state

### III EARTH & SATELLITE KINEMATICS

#### III.1 EARTH ORIENTATION

Earth Orientation here refers to the model for the orientation of the Earth-fixed reference relative to the Inertial reference. The former are necessary for associating observations, models and observatories to the geographic locations; and the latter for dynamics, integration & ephemerides.

Frame	System	Realization
Inertial	ICRS	J2000.0 (IERS)
Earth-fixed	CTRS	ITRF-2000

The rotation between the Inertial and Earth-fixed frames is implemented as:

$${}_{3 \times 3} M_{trs}^{crs} = PNRW$$

which converts the column array of components of a vector in the terrestrial frame to a column array of its components in the inertial frame. Each component matrix is itself a 3x3 matrix, and is now individually described.

Option 1 offered in the *IERS 96 Conventions* (Chapter 5) is implemented.

In the following,  $R_1, R_2, R_3$  refer to the elementary 3x3 rotation matrices about the principal directions X, Y and Z, respectively.

##### ***III.1.1 Precession (P)***

Following *IERS-96*, the IAU 1976 Precession is modeled as

$$P = R_3(\xi_A) R_1(-\theta_A) R_3(z_A)$$

where the component angles are evaluated using formulas in USNO Circular 163, Page A2. Reference epoch 2000.0 is used. The independent variable is TDT since epoch J2000.0 (noon, 01-Jan-2000).

##### ***III.1.2 Nutation***

Following *IERS-96*, the IAU 1980 Nutation model is used along with the associated corrections, such that

$$N = R_1(-\varepsilon_A)R_3(\Delta\psi)R_1(\varepsilon_A + \Delta\varepsilon_A)$$

The calculation of the nutation angles & their corrections is now summarized.

Quantity	Model	Notes
Obliquity of Ecliptic ( $\varepsilon_A$ )	Polynomial	USNO Circular 163, Page A3
Nutations in Longitude or Right Ascension ( $\Delta\psi$ ) & Obliquity ( $\Delta\varepsilon$ )	Interpolation of nutations in DE421	IAU 1980
Nutation Corrections	Planetary corrections: 25 largest terms	( <i>Souchay 1995</i> )
	Anelasticity not included	

### III.1.3 Sidereal Rotation (R)

This rotation is implemented as

$$R = R_3(-GST)$$

where the Apparent Greenwich Sidereal Time (GST) is calculated as follows:

Quantity	Model	Notes
Tabular variations	Cubic interpolation	<i>IERS C04</i>
	Diurnal tidal variations	Not modelled
	Nutation Corrections – 25 largest corrections to IAU 1980.	<i>IERS96</i>
GMST	Polynomial	USNO Circular 163, Page A3
Equatorial components of precession & nutation	( <i>Aoki &amp; Kinoshita</i> )	<i>IERS 96</i>
NOTE: Sidereal rotation rate is directly used in a single step GMST calculation, instead of the two-step calculation presented in <i>IERS-96</i> .		

### III.1.4 Wobble (W)

The Polar Motion component of rotation is implemented as

$$W = R_1(y_p)R_2(x_p)$$

Quantity	Model	Notes
Tabular variations	Cubic interpolation	<i>IERS C04</i>
Ocean Tidal Variations (Diurnal/Semi-Diurnal)	Not Modelled	
<p>Note 1: The rotation matrices are implemented in the small angle, skew-symmetric matrix formulation.</p> <p>Note 2: Rotational deformation accelerations &amp; kinematic station displacements are proportional to the difference between this time-series and a linear model for the pole.</p>		

### III.1.5 Rotation of Velocity Components

The position rotations are specified in Section II.1. The velocity components are rotated using the matrix approximation

$$\vec{v}_{crs} = M_{crs}^{trs} \vec{v}_{trs} + (PN\dot{R}S) \vec{r}_{trs}$$

## III. 2 STATION COORDINATES

This section summarizes the models for the mean and time-variable parts of the station coordinates adopted for data processing. It is important to understand that the JPL L-2 production fixes the GPS ephemerides to the JPL “FLINN” solution, and thus the station coordinates do not appear explicitly in the L-2 solution, but only implicitly in the FLINN solution.<sup>1</sup>

For the FLINN solution, the following standards are used:

Quantity	Model	Notes
Mean Station Positions	IGS08	Refers to the position of a geodetic marker and reference point for antenna calibrations. IGS realization of ITRF2008
Station Velocities	Individual Station velocities in ITRF2008	
Station Eccentricities	See individual observation models models and IGS08 antenna calibrations	
Ocean Tidal Loading	FES2004 with hardisp.f	Spline interpolation of 342 constituents from 11 tides

<sup>1</sup> Desai, S. D., W. Bertiger, J. Gross, B. Haines, N. Harvey, C. Selle, A. Sibthorpe, and J. P. Weiss, *Results from the Reanalysis of Global GPS Data in the IGS08 Reference Frame*, EOS Trans, AGU, 2011.

Luni-Solar Solid Earth Tidal Displacement	IERS2010 Standard	Luni-Solar ephemerides from DE-421
Rotational Deformation	IERS2010 Standard	Cubic mean pole model.
Ocean Pole Tide Loading	IERS2010 Standard	
Tidal Geocenter Correction	Not modeled	
$S_1$ - $S_2$ Atmospheric Loading	Not modeled	
Post-glacial Rebound	Not modeled	
Slow (seasonal) Geocenter Variations	Not modeled	

### III. 3 SATELLITE KINEMATICS

The inertial orientation of the spacecraft is modeled using tabular input data quaternions. The same data (with appropriate definitions) is used for rotating the accelerometer data to inertial frame prior to numerical integration; for making corrections to the ranging observations due to offset between the satellite center of mass & the antenna location; as well as for computing the non-gravitational forces (if necessary).

At epochs where the GRACE quaternion product is not available, linear interpolation between adjacent values is used.